Question 2

1. The first regulation:
   1. 8-hour pay: fixed costs as all attendants who work will be paid
   2. Overtime pays:
      1. yi1 = 1 if attendant i works the first overtime hour
      2. yi2 = 1 if attendant i works the second overtime hour
      3. To ensure proper ordering: y₂ᵢ ≤ y₁ᵢ (can't work 2nd overtime hour without working 1st)

The second regulation:

1. Floor assignment: # of floors the attendants are assigned
   * Zi3 = 1 if attendant i works three floors in total
   * Zi4 = 1 if attendant i works four floors in total
   * To ensure proper ordering: Zi4 ≤ Zi3

The cost function would then include:

Base pay: 8 \* $25 per attendant

Overtime: 1.5 \* $25 \* (y₁ᵢ + y₂ᵢ) per attendant

Extra floor bonus: $75 \* (z₁ᵢ + z₂ᵢ) per attendant

Let xijr = 1 if attendant i cleans room r on floor j, 0 otherwise

Let fik = 1 if attendant i works on floor k for a minimum of one room, 0 otherwise

The constraints should be:

∑(xijr) ≥ fik (for each attendant i and floor k)

∑(xijr) ≤ M × fik

Where:

* ∑(xijr) is the sum of all rooms on floor k assigned to attendant i
* M is the total number of rooms on floor k
* The upper bound M × fik prevents assigning more rooms than exist on the floor

1. It is a non-linear constraint with a cost penalty. Because it has a threshold of 3500 sqft, when it triggers, it leads to double hourly pay for each attendant; it also requires a binary variable to track if the threshold is exceeded. Like below:

∑(square\_feet × xijr) ≤ 3500 + M × si

Where:

• si = 0  means the attendant stays **within 3,500 sqft**.

• si = 1  means the attendant **exceeds 3,500 sqft** and triggers double pay.

1. If the cost penalties have to be incurred, then the hotel should prioritize penalties in the following order:

1. **Exceeding 3,500 sqft → Most Expensive**

• If an attendant cleans **more than 3,500 sqft**, their wage **doubles** for the extra square footage.

• This creates the highest penalty because the **extra wage applies to every additional square foot** cleaned.

2. **Overtime (Up to 2 Hours at 1.5x Pay) → Medium Cost**

• If an attendant works more than **8 hours**, they are **paid 1.5× their wage** for the extra time.  
• Since overtime is capped at **2 hours**, the total penalty is more predictable than square footage penalties.

1. **Exceeding 2 Floors → Least Expensive**

• If an attendant cleans **more than two floors**, they get a **flat $75 per extra floor**.

• This penalty is fixed per floor and does not scale with time or area, making it less costly than the others.

If overtime is paid at 2× instead of 1.5×, it will become more expensive than the floor penalty but still cheaper than the square footage penalty. The new priority order is below (cost from high to low):

1. Exceeding 3,500 sqft (most expensive, applies to working hours for all excess area cleaned).

2. Overtime (now at 2×, making it more expensive than floor penalties).

3. Exceeding 2 floors ($75 per extra floor, still the least expensive).

1. A screenshot of a computer

   AI-generated content may be incorrect.
2. A screen shot of a black screen

   AI-generated content may be incorrect.
3. A screenshot of a computer

   AI-generated content may be incorrect.

Key Observations:

1. Cost Difference:
   * The linear relaxation provides a lower cost solution
   * Cost gap: $89.49 (approximately 5.11% lower)
2. Overtime Hours:
   * Binary program: 4.0 discrete overtime hours
   * Linear relaxation: 1.61 fractional overtime hours

Implications:

1. Lower Bound Approximation: The linear relaxation provides a lower bound on the optimal cost. This is expected because:
   * Continuous variables allow more flexible assignments
   * Relaxing integer constraints removes some optimization constraints
2. Not a Direct Approximation: The relaxed solution cannot be directly used as a practical solution because:
   * Fractional room/overtime assignments are not feasible in reality
   * Rounding the relaxed solution may not yield a valid or optimal integer solution
3. Solution Quality:
   * The relatively small gap (5.11%) suggests the linear relaxation provides a reasonable lower bound
   * However, it does not capture the exact resource allocation of the binary program
   1. Both methods gave exactly the same optimal cost ($1660.51)
   2. Both methods produced identical solutions for room assignments, overtime hours, and floor assignments
   3. Both methods resulted in the same fractional values for the decision variables

The only difference is in implementation approach, not in the mathematical results. This illustrates an important principle in optimization: different implementation methods that represent the same mathematical program will yield the same optimal solution

1. This validates the intuition from part d:
   1. When overtime became more expensive (2×), the model preferred a floor violation (which we didn't see in the 1.5× case)
   2. No square footage violations occurred in either case, confirming it remains the most expensive
   3. The model reduced overtime hours from 4 to 2 when the rate doubled